Analysis of Variance for Designed Experiments

Question 1. The effects of sulfone concentration and surface temperature on the mean surface reflectivity are being studied in a 2-factor factorial experiment. The results are shown in the "Surface Condition Dataset 1". (For this question only use x1 = sulfone concentration, x2 = surface temperature, y = surface reflectivity)

a) Write a complete regression model for this factorial experiment.

The regression equation is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_1 x_2 + \beta_7 x_1 x_3 + \beta_8 x_1 x_4 + \beta_9 x_1 x_5$$

x_1 = 1 if the concentration level is 5,

0 otherwise

x_2 = 1 if the temperature is 75,

0 otherwise

x_3 = 1 if the temperature is 100,

0 otherwise

x_4 = 1 if the temperature is 125,

0 otherwise

x_5 = 1 if the temperature is 150,

0 otherwise

$$E(Y) = 33.4 - 4.2x_1 + 5.767x_2 + 3.767x_2 + 0.433x_4$$
$$- 2.9x_5 + 1.7(x_1 x_2) + 1.7(x_1 x_3) + 1.7(x_1 x_4) \; - 2.633(x_1 x_5) + \varepsilon_{ijk}$$

and ε_{ijk} is a random error component. Also E(ε_{ijk}) = 0 and V(ε_{ijk}) = σ^2 (Constant across all levels)

b) Do the data present sufficient evidence to indicate an interaction between sulfone concentration and surface temperature (use α = 0.01)?

My work and software outputs start on page	**Page # 1**
b.1) The Null Hypothesis is	$H_0: \beta_6 = \beta_7 = \beta_8 = \beta_9 = 0$
b.2) The rejection region is	$F_{1,\beta}$ > 4.579
b.3) My answer is	Do not reject the null hypothesis at 1 % level. (F-value = 3.02)

c) Is it appropriate to conduct tests for main effects? (Use α = 0.01)

My answer is (Yes/No)?	**Yes**
Why (a single sentence)?	The hypotheses concerned with the main effects are rejected.

c.1) If yes (no), what is (would have been) the null hypotheses for testing sulfone concentration	$H_0: \beta_1 = 0$
c.2) If yes (no), what is (would have been) the null hypotheses for testing surface temperature	$H_0: \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$

d) Generate an ANOVA summary table and an Interaction Plot

My software outputs (ANOVA table) start on page	**Page 2**
Which statistic in the ANOVA table concurs with your answer in b.3)?	The F statistic concerned with the interaction effect in the ANOVA Table concurs with my answer in b.3
What levels of the two factors result in the highest mean surface reflectivity?	x1 = 2; x2 = 1 y = 41.67

Question 2. The effects of sulfone concentration and surface temperature on the mean surface reflectivity are being studied in a 2-factor factorial experiment. The results are shown in the "Surface Condition Dataset 2". (Suppose x2 = surface temperature, y = surface reflectivity). In each question, when the concentration is 5 grams/liter, highlight (like this) the correct statement. (Use α = 0.01)

My work and software outputs start on page	Page 7

a) The mean surface reflectivity is significantly higher when the x2 = 150 than when x2 = 75 The mean surface reflectivity is significantly lower when the x2 = 150 than when x2 = 75 The mean surface reflectivity when the x2 = 150 does not significantly differ from the mean surface reflectivity when the x2 = 75.

b) The mean surface reflectivity is significantly higher when the x2 = 125 than when x2 = 175 The mean surface reflectivity is significantly lower when the x2 = 125 than when x2 = 175 The mean surface reflectivity when the x2 = 125 does not significantly differ from the mean surface reflectivity when the x2 = 175.

c) The mean surface reflectivity is significantly higher when the x2 = 100 than when x2 = 75 The mean surface reflectivity is significantly lower when the x2 = 100 than when x2 = 75 The mean surface reflectivity when the x2 = 100 does not significantly differ from the mean surface reflectivity when the x2 = 75.

Use the table below for Questions 3 through 5

Technician 1	Technician 2	Technician 3	Technician 4
9	14	6	10
12	9	14	12
8	12	10	7
10	10	8	15
11	14	11	11

Question 3. The table above shows the number of mistakes made in 5 successive days for 4 technicians (these are not necessarily the same 5 days for all 4 technicians). Is there sufficient evidence to indicate a difference in the mean number of mistakes by the 4 technicians? (Use α = 0.01)

My work and software outputs start on page # 8

3.a) The regression equation is $E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

x_1 = 1 if the Technician 1,

0 otherwise

x_2 = 1 if the Technician 2,

0 otherwise

x_3 = 1 if the Technician 3,

0 otherwise

$$E(Y) = 10.65 + -0.65x_1 - 1.15x_2 - 0.85x_3$$

3.b) The null hypothesis is $H_0: \beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$

3.c) The rejection region is $F_{Technician} > 5.292$ (at 1 % level)

3.d) The conclusion is : Since the calculated F ratio is less than the F—critical value (5.292), we conclude that the sample does not provide enough evidence to reject the null hypothesis. That is there is no significant difference among the four technicians with respect to the number of mistakes done by them.

Question 4. Now suppose the experimenter had instead decided the record the number of mistakes made during the same 5 successive days for the 4 technicians. Is there sufficient evidence to indicate that recording the numbers for the same 5 days for all 4 technicians made a difference? (Use α = 0.01).

My work and software outputs start on page # 8

4.a) The regression equation is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7$$

x_1 = 1 if the Technician 1,

0 otherwise

x_2 = 1 if the Technician 2,

0 otherwise

x_3 = 1 if the Technician 3,

0 otherwise

x_4 = 1 if it is day 1 ,

0 otherwise

x_5 = 1 if it is day 2 ,

0 otherwise

x_6 = 1 if it is day 3 ,

0 otherwise

x_7 = 1 if it is day 4,

0 otherwise

$$E(y) = 10.65 - 0.65x_1 + 1.15x_2 - 0.85x_3 - 0.9x_4 + 1.1x_5 - 1.4x_6 + 0.1x_7$$

4.b) The null hypothesis is $H_{0T}: \beta_0 = \beta_1 = \beta_2 = \beta_3$

$$H_{0D}: \beta_4 = \beta_5 = \beta_6 = \beta_7 = 0$$

4.c) My ANOVA Table is on page # 8

4.d) My answer is Reject both the Null hypotheses, as their corresponding p- values are greater than 0.01.

Question 5. Referring to Question 4, is there sufficient evidence to indicate a difference in the mean number of mistakes by the 4 technicians? (Use α = 0.01).

My work and software outputs start on page # 8

5.a) The regression equation is

$$E(y) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7$$

x_1 = 1 if the Technician 1,

0 otherwise

x_2 = 1 if the Technician 2,

0 otherwise

x_3 = 1 if the Technician 3,

0 otherwise

x_4 = 1 if it is day 1 ,

0 otherwise

x_5 = 1 if it is day 2 ,

0 otherwise

x_6 = 1 if it is day 3 ,

0 otherwise

x_7 = 1 if it is day 4, 0 otherwise

$$E(y) = 10.65 - 0.65x_1 + 1.15x_2 - 0.85x_3 - 0.9x_4 + 1.1x_5 - 1.4x_6 + 0.1x_7$$

5.b) The null hypothesis is

$$H_{0T}: \beta_0 = \beta_1 = \beta_2 = \beta_3$$

$$H_{0D}: \boldsymbol{\beta}_4 = \boldsymbol{\beta}_5 = \boldsymbol{\beta}_6 = \boldsymbol{\beta}_7 = 0$$

5.c) My answer is : Since both the p-values corresponding to these null hypothesis are greater than 0.01, we conclude that there is no significant difference among the four Technicians and also among the five days considered.