

The manager at Trion Electric Co-op is concerned about the rise in the number of late payments. If more than 10% of the accounts are behind in their payments, the manager will initiate a costly monitoring program. A random sample of 75 accounts shows that 9 were behind in their payments.

a. Does the sample data provide evidence to conclude that more than 10% of all accounts are delinquent (using $\alpha=0.01$)?

1. Formulate the null and alternative hypotheses.

Solution:

a. Does the sample data provide evidence to conclude that more than 10% of all accounts are delinquent (using $\alpha=0.01$)?

The information given in the above problem can be represented with the following notations.

Sample size = $n = 75$

$P_0 = 0.1$

$Q_0 = 0.9$ (since $Q_0 = 1 - P_0$)

The number of persons behind their payments is $x = 9$

$$\hat{p} = \frac{x}{n} = \frac{9}{75} = 0.12$$

Significance level = 0.01

1. Null hypothesis:

$$H_0: P \leq 0.1$$

That is, not more than 10 % of all accounts are delinquent

Alternate hypothesis:

$$H_1: P > 0.1$$

That is, more than 10 % of all accounts are delinquent

2. **Criterion for rejection of the null hypothesis:**

If the P-value of the test statistic is less than 0.05 we reject the null hypothesis.

3. **Test statistic:**

$$Z = \frac{\hat{p} - P_0}{\sqrt{\frac{P_0 * Q_0}{n}}}$$

$$= \frac{0.12 - 0.1}{\sqrt{\frac{0.1 * 0.9}{75}}}$$

$$= \frac{0.02}{\sqrt{0.0012}}$$

$$= \frac{0.02}{0.034641}$$

$$= 0.57735$$

Hence the test statistic value is 0.57735

4. P-value:

$$\text{P-value} = P [Z > 0.57735]$$

$$= 1 - P [Z < 0.57735]$$

$$= 1 - 0.718148$$

$$= 0.281852$$

Hence the P- value of the test statistic is 0.281852

5. Statistical decision:

Since the P-value of the test statistic is greater than the significance level (that is $0.281852 > 0.01$) we conclude that there is no enough evidence to reject the null hypothesis at 0.01 level. Thus we conclude that not more than 10 % of all accounts are delinquents.